Fractal analysis of ammonoid septa

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ABSTRACT : modern digital processing techniques bring new tools to ammonoid research.

Introduction

Recent strides in mathematics led to a new understanding of natural processes, especially those related to development of living entities. It is particularly easy to notice mathematics in living forms like trees, plants such as cauliflowers and even mineral forms (snow flakes, crystals, etc...). All of them have in common self similar structures, usually refered as fractals.

Non heteromorph ammonoids are fractals in two ways : their shell grows in a logarithmic way and if one zooms the ombilicus of an ammonoid, one will see something ressembling very closely the whole specimen.

The second way is even more interesting since it can be applied to heteromorph ammonoids as well. Many authors were surprised by the apparent complexity of suture patterns of ammonoids. Less were interested in studying them upon a mathematic point of vue. The aim of this paper is to show the reason why ammonoid septa are fractals and the usefulness of simple and less simple mathematic tests and computer algorithms for ammonoid research.

Explaining the word "fractal"

Between the fifties and the seventies, Benoît Mandelbrot developed a new branch of mathematics which is able to describe and analyze the structured irregularity of the natural world. He invents the word "fractal" to describe objects which have a very detailled structure on multiple scales.

A fractal is a geometric shape whose details are quite similar to the global shape. That is why people usually refer to self-similar structures.

One of the most famous fractals is the Von Koch curve. The figure below shows three steps of the building of this curve. The first picture show the basic pattern. The principle consists in copying the basic pattern on each segment of itself and reiterate this process (in true fractals, this process is infinite). The second and third pictures show the result after one and two iterations.

Fractal dimension of a suture line

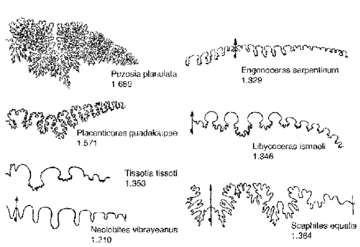
In fractal geometry, there exist splines which are so curved on themselves that they tend to generate a space of dimension 2. On the other side, simple lines or splines dimension tends to 1.

We can compute the value of the fractal dimension of the Von Koch curve just by counting the total number of segments and the number of segments of the shortest path between the two ends :

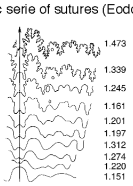
$$d = \log(4) / \log(3) \approx 1.262$$

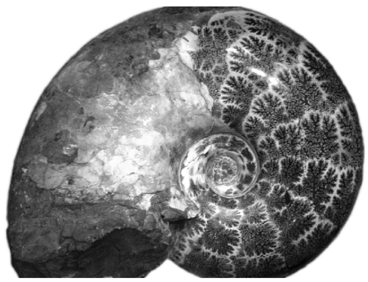
Finding the fractal dimension of the suture of any ammonoid specie is quite easy and provides an interesting result which quantifies its complexity.

The list below provides results of this technique applied to some of the most typical suture classes :



The picture below (modified after Mikhailova [2]) provides the fractal dimension of an ontogenic serie of sutures (Eodouvilleiceras CASEY).





Pachydiscus catarinae ANDERSON & HANNA

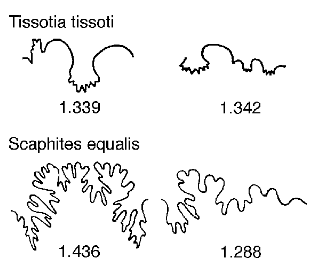
Since the fractal dimension is a quantifying criteria, it would be interesting to use it as a basic criteria for checking the accuracy of the current classification of ammonoids. Yet, this technique is not enough : one can see clearly that quite similar results can be obtained with sutures of ammonoids which are not close to each other in the systematics. A perfect example is the similar fractal dimension of Tissotia tissoti and Scaphites equalis which belong to different sub-orders.

Multiple fractal dimensions

It is possible to obtain better results by using a slightly different method : by considering the suture line segment by segment. If one reconsiders the case of T. tissoti and S. equalis this way, the results will be different and more interesting.

In the example below, the sutures are divided into two segments. The fractal dimension is computed for each of them. In the first example, T. tissoti, one can notice that the dimension of both segments is nearly equal to the dimension of the whole suture (which shows that the complexity is quite constant). The second example shows an obvious difference : the segments are different in complexity, which is easy to notice both visually and on a mathematic point of vue.

The previous method could not emphasize this difference since the suture was considered in its whole.



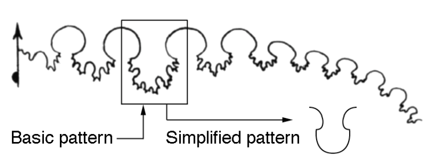
Partial conclusion : the simple fact of considering a suture in its whole is not enough to provide a quantifying criteria capable of making the difference between two ammonoids.

Modelizing sutures

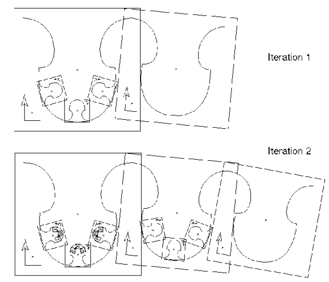
We need to take in account all features of the sutures to be able to make the difference between ammonoids which obviously do not belong to the same branch.

To do that, it is necessary to understand how suture patterns are generated. The best thing to do is to try to generate patterns similar to them. Generating such patterns can be achieved using the same technique as the one used to generate the Von Koch curve.

In order to try to generate something realistic, a good method consists in using the basic pattern (also called axiom) of a real suture. In this case, Libycoceras ismaeli is a good choice because this pattern is quite easy to find.



Once the axiom is found, one has to find an IFS (Iterated Function System) which is able to produce patterns which imitate the suture. Figure below (Iteration 1) shows the chosen IFS. The basic pattern is repeated five times after rotation and scaling.



The second figure (Iteration 2) shows the result after repeating this process a second time.

Partial conclusion : it is possible to mimic ammonoid sutures by using simple patterns and elaborated IFS. Although the result is different, it is enough to understand the process which leads to such patterns and that sutures can be considered as being fractal.

Reverse engineering

We saw previously that the very small details of sutures come from the way they (sutures) are generated. Finding a comprehensive model is possible by using reverse engineering techniques in order to find the axiom and the IFS which lead to a given suture.

At the moment, there is no satisfying results, yet a technique being developed (not described in this paper) could provide results in the next future.

Conclusion

Provided these methods are improved in the near future, they could rapidly bring tools capable of accurate analysis of ammonoid sutures :

• Checking the systematics, especially in the case of doubtful or debatable phylogeny.

• Using such techniques in association with "in-situ" collectings could help differenciating :
  • species which look closely like each other
  • sub-species, varieties and "transitional specimens"
This could maybe give precise details about biostratigraphy and increase its accuracy.

• One of the most fascinating tools based upon these techniques would be a tool capable of making attempts of identifying ammonoid specimens by analyzing their sutures. In order to carry out this project, it is necessary to build a comprehensive database of sutures. Such a work would require a team of both people specialized in computing and ammonoid systematics/phyllogeny.

• Finding the axiom and IFS which generate all the ammonoids could be useful to try to understand the origin of Ceratites-like sutures in Cretaceous ammonoids (Engonoceratidae HYATT, Coilopoceratidae HYATT, Sphenodiscidae HYATT, etc...)

• Trying to correlate the complex morphology of septa and the hydrostatic pressure resistance for a given ammonoid.

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